Supplementary Results: Extension to Multiple-Bit Case

A. Analyses

Assume that B bits ($B \geq 1$) are used at each relay for SNR quantization. Assume also that, at the ith relay, the B bits, denoted by $\{q_{i,1}, q_{i,2}, \cdots, q_{i,B}\}$, are then transmitted with the on-off signaling over B parallel BSC's each with crossover probability p_i, $1 \leq i \leq L$. Denote by $\hat{q}_{i,j}$ the jth received bit from the ith relay, $1 \leq i \leq L$ and $1 \leq j \leq B$. Let us collect all $\hat{q}_{i,j}$'s into a matrix to form

$$\widehat{\mathbf{Q}} = \begin{bmatrix} \hat{q}_{1,1}, \cdots, \hat{q}_{1,B} \\ \hat{q}_{2,1}, \cdots, \hat{q}_{2,B} \\ \vdots \qquad \vdots \\ \hat{q}_{L,1}, \cdots, \hat{q}_{L,B} \end{bmatrix}, \tag{1}$$

in which the ith row consists of the received B-bit message of ith S-R link SNR. For a given $\widehat{\mathbf{Q}}$, let us collect all $\widetilde{\mathbf{Q}} = \begin{bmatrix} \tilde{q}_{1,1}, \cdots, \tilde{q}_{1,B} \\ \tilde{q}_{2,1}, \cdots, \tilde{q}_{2,B} \\ \vdots \qquad \vdots \\ \tilde{q}_{L,1}, \cdots, \tilde{q}_{L,B} \end{bmatrix}$'s that differ from $\widehat{\mathbf{Q}}$ in exactly l rows to obtain

$$S_l(\widehat{\mathbf{Q}}) = \left\{ \widetilde{\mathbf{Q}} \,\middle|\, \sum_{i=1}^{L} u(a_i) = l, \text{ where } a_i = \sum_{j=1}^{B} \hat{q}_{i,j} \oplus \tilde{q}_{i,j} \text{ and } u(\cdot) \text{ is the unit-step function} \right\}. \tag{2}$$

In addition, associated with each $\widetilde{\mathbf{Q}} \in S_l(\widehat{\mathbf{Q}})$ we define

$$I_l(\widehat{\mathbf{Q}}, \widetilde{\mathbf{Q}}) = \left\{ (i,j) \,\middle|\, \tilde{q}_{i,j} \neq \hat{q}_{i,j} \right\}, \tag{3}$$

which consists of the locations of all different entries between $\widetilde{\mathbf{Q}}$ and $\widehat{\mathbf{Q}}$. Based on (2) and (3), the conditional average SNR $\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h}_r, \widehat{\mathbf{Q}})$ in the multiple-bit case is obtained as

$$\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h}_r, \widehat{\mathbf{Q}}) = \sum_{l=0}^{L} \sum_{\widetilde{\mathbf{Q}} \in S_l(\widehat{\mathbf{Q}})} \Pr(\widetilde{\mathbf{Q}} | \widehat{\mathbf{Q}}) \gamma_{dq}(\mathbf{g}, \mathbf{h}_r, \widetilde{\mathbf{Q}}), \tag{4}$$

where $\Pr(\widetilde{\mathbf{Q}} | \widehat{\mathbf{Q}}) = \left(\prod_{(i,j) \in I_l(\widehat{\mathbf{Q}}, \widetilde{\mathbf{Q}})} p_i \right) \left(\prod_{(i,j) \in I_l^c(\widehat{\mathbf{Q}}, \widetilde{\mathbf{Q}})} (1 - p_i) \right)$ and $I_l^c(\widehat{\mathbf{Q}}, \widetilde{\mathbf{Q}})$ is the complement of $I_l(\widehat{\mathbf{Q}}, \widetilde{\mathbf{Q}})$. By following the same techniques as in the paper and through further manipulations, $\overline{\gamma}_{dq}(\mathbf{g}, \mathbf{h}_r, \widehat{\mathbf{Q}})$ can be further expressed as

$$\overline{\gamma}_{dq}(\mathbf{g},\mathbf{h}_r,\widehat{\mathbf{Q}}) = \sum_{l=0}^{L}\left(\sum_{k_1=1}^{L}\sum_{k_2=k_1+1}^{L}\cdots\sum_{k_l=k_{l-1}+1}^{L}\sum_{j_1=1}^{\tau}\sum_{j_2=1}^{\tau}\cdots\sum_{j_l=1}^{\tau}\frac{\left|\sum_{i=1}^{L}c_i(l,k_1,\cdots,k_l,j_1,\cdots,j_l)g_i\right|^2}{\sum_{i=1}^{L}\left|g_i\right|^2 d_i(l,k_1,\cdots,k_l,j_1,\cdots,j_l)}\right), \tag{5}$$

where

$$\tau = 2^B - 1\,, \tag{6}$$

$$c_i(l,k_1,\cdots,k_l,j_1,\cdots,j_l) \triangleq \sqrt{\eta\prod_{t=1}^{l}p_{k_t}^{v_t}\left(\prod_{t=1}^{l}(1-p_{k_t})^{v_t}\right)^{-1}}\,h_{r,i}\phi(\mathbf{r}_i)\,, \tag{7}$$

$$d_i(l,k_1,\cdots,k_l) \triangleq \left|h_{r,i}\right|^2\left[1-\phi^2(\mathbf{r}_i)\right]+\frac{\sigma_w^2}{P_d}, \tag{8}$$

$$\eta \triangleq \prod_{l=1}^{L}(1-p_l)^B\,, \tag{9}$$

$$\mathbf{r}_i \triangleq \begin{cases}\left[\hat{q}_{i,1}\cdots\hat{q}_{i,B}\right],\ i\neq k_1,\cdots,k_l\\ \left[\hat{q}_{i,1}^t,\hat{q}_{i,2}\cdots\hat{q}_{i,B}\right],\ j_t=1,\ i=k_t,\ t\in\{1,\cdots,l\}\\ \left[\hat{q}_{i,1},\hat{q}_{i,2}^t,\hat{q}_{i,3}\cdots\hat{q}_{i,B}\right],\ j_t=2,\ i=k_t,\ t\in\{1,\cdots,l\}\\ \quad\vdots\\ \left[\hat{q}_{i,1},\hat{q}_{i,2}\cdots\hat{q}_{i,B}^t\right],\ j_t=C_1^B,\ i=k_t,\ t\in\{1,\cdots,l\}\\ \left[\hat{q}_{i,1}^t,\hat{q}_{i,2}^t,\hat{q}_{i,3}\cdots\hat{q}_{i,B}\right],\ j_t=C_1^B+1,\ i=k_t,\ t\in\{1,\cdots,l\}\\ \left[\hat{q}_{i,1}^t,\hat{q}_{i,2},\hat{q}_{i,3}^t,\hat{q}_{i,4},\cdots\hat{q}_{i,B}\right],\ j_t=C_1^B+2,\ i=k_t,\ t\in\{1,\cdots,l\}\\ \quad\vdots\\ \left[\hat{q}_{i,1}^t,\hat{q}_{i,2}\cdots\hat{q}_{i,B-1},\hat{q}_{i,B}^t\right],\ j_t=C_1^B+B-1,\ i=k_t,\ t\in\{1,\cdots,l\}\\ \left[\hat{q}_{i,1},\hat{q}_{i,2}^t,\hat{q}_{i,3}^t,\hat{q}_{i,4},\cdots\hat{q}_{i,B}\right],\ j_t=C_1^B+B,\ i=k_t,\ t\in\{1,\cdots,l\}\\ \left[\hat{q}_{i,1},\hat{q}_{i,2}^t,\hat{q}_{i,3},\hat{q}_{i,4}^t,\hat{q}_{i,4}\cdots\hat{q}_{i,B}\right],\ j_t=C_1^B+B+1,\ i=k_t,\ t\in\{1,\cdots,l\}\\ \quad\vdots\end{cases}, \tag{10}$$

where $\hat{q}_{i,i'}^t = \hat{q}_{i,i'}\oplus 1$, and $v_t = n$, where $\sum_{i=0}^{n-1}C_i^B \le j_t \le \sum_{i=1}^{n}C_i^B$. Through further rearranging the indices in the multiple summations in (5), $\overline{\gamma}_{dq}(\mathbf{g},\mathbf{h}_\mathbf{r},\widehat{\mathbf{Q}})$ can be expressed as a single sum of Rayleigh quotients, which is shown as

$$\overline{\gamma}_{dq}(\mathbf{g},\mathbf{h}_r,\widehat{\mathbf{Q}}) = \sum_{m=1}^{M}\frac{\left|\mathbf{c}_m^H\mathbf{g}\right|^2}{\mathbf{g}^H\mathbf{D}_m\mathbf{g}}, \tag{11}$$

where

$$\mathbf{c}_m^H \triangleq [c_1(l,k_1,\cdots,k_l,j_1,\cdots,j_l),\cdots,c_L(l,k_1,\cdots,k_l,j_1,\cdots,j_l)], \tag{12}$$

$$\mathbf{D}_m \triangleq diag\left\{d_1(l,k_1,\cdots,k_l,j_1,\cdots,j_l),\cdots,d_L(l,k_1,\cdots,k_l,j_1,\cdots,j_l)\right\}, \tag{13}$$

$$m = \delta(l) + \sum_{s_0=0}^{l-1} C_{s_0}^{L} + \sum_{\lambda=1}^{l} \sum_{s_\lambda=k_{\lambda-1}+1}^{k_\lambda-1} C_{l-\lambda}^{L-s_\lambda} + \sum_{\lambda_2=1}^{l} \tau^{l-\lambda_2}(j_{\lambda_2}-1)+1, \quad (14)$$

and $M = \sum_{i=0}^{L} C_i^L \tau^i$, where τ is defined in (6).

B. Simulation Results

For the two-bit case, i.e., $B=2$, computer simulation is conducted to compare the BER performances of the proposed method and the solution in [18]. In the simulation setup, the number of relay nodes is $L=4$, the average SNR of the S-R link is set to be $\overline{\gamma}_s = 20$ dB, and the crossover probability p_i of each BSC follows the uniform distribution over the interval $[0.05, 0.1]$. The simulated BER with respect to different R-D link SNR is then plotted in Figure A.1, shown below. It can be seen from the figure that, as expected, the proposed method outperforms the solution in [18].

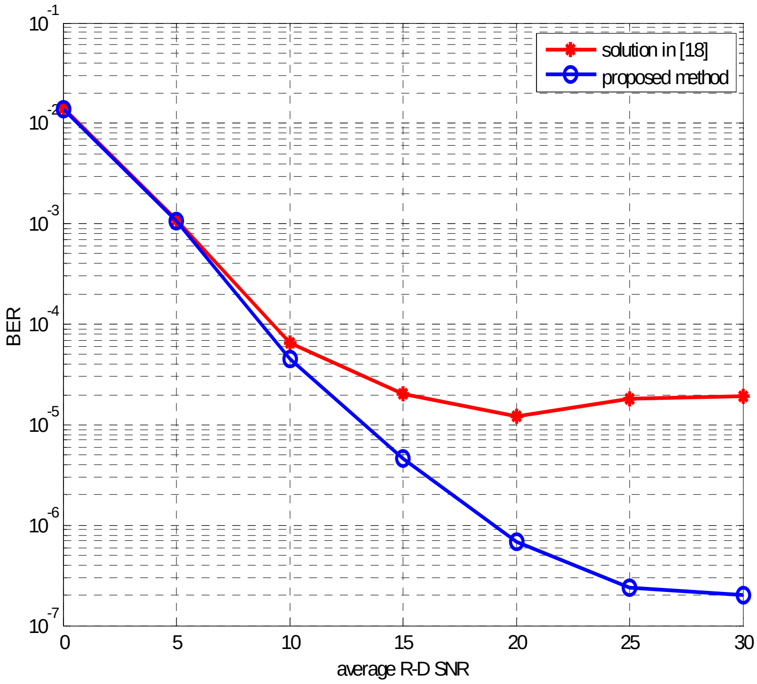

Figure A.1. BER results for two methods when 2-bit SNR quantization is adopted at each relay.